W5 ... VI

UNITED STATES DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
NATIONAL WEATHER SERVICE
Silver Spring, Md. 20910

February 12, 1979 OA/W111x1/RER

TO: Recipients of Technical Procedures Bulletin Series

FROM: Duane S. Cooley Duane S. Cooley
Chief, Technical Procedures Branch
Meteorological Services Division, NWS

SUBJECT: Technical Procedures Bulletin No. 258: THE NATIONAL METEOROLOGICAL CENTER'S NINE-LAYER GLOBAL PRIMITIVE EQUATION MODEL

This bulletin was prepared by Mr. R. E. Rieck, Technical Procedures Branch, NWSH, from information supplied by Dr. John Stackpole of NMC. It describes in relatively brief form the current 9-Layer Primitive Equation model in operation at NMC.

This bulletin supersedes Technical Procedures Bulletin No. 115 which is now considered as operationally obsolete and as such may be removed from the station bulletin files.

THE NATIONAL METEOROLOGICAL CENTER'S NINE-LAYER GLOBAL PRIMITIVE EQUATION MODEL

1. INTRODUCTION

The NMC numerical modeling program is pointed in three main directions: (1) regional modeling, which includes limited-area fine-mesh models, planetary boundary layer models, and hurricane models; (2) hemispheric modeling; and (3) global modeling, which presently includes the global version of the 9-layer model. This bulletin concerns the latter type of system--specifically, the 9-layer prediction model system.

The primary motivations for global modeling for operational forecasting are to enlarge the area of validity of the forecasts and to extend the time period of significant skill for a given region, such as the United States. User requirements, as in the field of aviation forecasting and in the field of extended-range forecasting, have demanded a global modeling capability. Such prediction systems are planned to be fed to a large extent by the satellite observing systems. Research efforts have been, and are still being directed at obtaining better methods of incorporating all types of data into a system which will produce more accurate forecasts. A basic feature of the 9-layer prediction model which allows for the global forecast characteristic in the latitude-longitude grid (2.5°) with a finite-differencing system which amounts for the mathematical singularities, the poles.

This essay describes, in outline, the basic physics, structure, and design of the new NMC 9L GLO forecast model. Where appropriate differencies between the 9-layer and 7-layer (7L PE) models are indicated.

At the present time the 9L GLO is used safely in the Optimum Interpolation analysis system.

2. BASIC EQUATIONS

The fundamental physics are the same as in the 7L PE model, since they all use the hydrostatic (primitive) equations of motion derived from Newton's laws of motion. The new 9L PE will forecast these quantities:

a. Wind components u and v
b. Potential temperature θ
c. Pressure thickness P_σ. This is the difference in pressure between two arbitrary sigma(σ) surfaces. The definition of σ will be given later.
d. Specific humidity q. (The 7L PE uses precipitable water W.)

Each of these five quantities are assumed to describe the average conditions in nine suitably defined layers of the model atmosphere.

The equations are written in spherical coordinates (the 7L PE model uses Cartesian coordinates) which thus introduce some new terms into the equations and slightly change the meaning of u and v. If λ represents longitude and is defined as increasing to the east (locations will be referenced as east of the Greenwich meridian in the models), ϕ represents the latitude and is defined as increasing northward, and r represents the radius of the spherical earth, then

$$u \equiv r \cos \phi \, \frac{d\lambda}{dt}$$

$$v \equiv r \, \frac{d\phi}{dt}$$

i.e., u is the west wind (when positive) and v is the south wind (when positive).

The equations are also written in terms of an arbitrary vertical coordinate σ. One may speak of surfaces of constant σ (just like one speaks of surfaces of constant pressure or height) along which other meteorological quantities can and usually do vary. The precise definition of σ is given later.

The wind component equations are

$$\frac{\partial u}{\partial t} + \dot{\sigma}\frac{\partial u}{\partial \sigma} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{uv}{r}\tan\phi$$

$$- fv + g\frac{\partial z}{\partial x} + c_p\theta\frac{\partial \pi}{\partial x} + F_x = 0 \qquad (1)$$

and

$$\frac{\partial v}{\partial t} + \dot{\sigma}\frac{\partial v}{\partial \sigma} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{u^2}{r}\tan\Phi$$

$$+ fv + g\frac{\partial z}{\partial y} + c_p\theta\frac{\partial \pi}{\partial y} + F_y = 0 \qquad (2)$$

In these equations, $\dot{\sigma}$ is the vertical velocity and the terms involving it are the measures of vertical advection; f is the Coriolis parameter $2\Omega\sin\Phi$ (Ω = earth's rotation rate) accounting for the rotation of our coordinate system; c_p is the specific heat of air at constant pressure; $\pi = (p/1000)^{R/c_p}$ (p is in mb, R is the universal gas constant) is known as the Exner function and is no more than a convenient measure of pressure; g is gravitational acceleration and z is the height above mean sea level of the sigma layer in which the variables are being forecast. In terms of the actual longitude-latitude coordinates

$$\frac{\partial}{\partial x} = \frac{\partial}{r\cos\Phi\,\partial\lambda}\ ;\ \frac{\partial}{\partial y} = \frac{\partial}{r\partial\Phi}$$

i.e., the east-west or north-south increment of distance on the earth for a given increment of longitude or latitude. These derivatives are, of course, taken along surfaces of constant σ (that's the meaning of the partial derivative notation).

The first lines of equations (1) and (2) are the advective and metric terms. the second lines are the forces involved: Coriolis, height and pressure gradient, and friction.

The temperature and specific humidity tendency equations are, respectively

$$\frac{\partial \theta}{\partial t} + \dot{\sigma}\frac{\partial \theta}{\partial \sigma} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + HC = 0 \qquad (3)$$

and

$$\frac{\partial q}{\partial t} + \dot{\sigma}\frac{\partial q}{\partial \sigma} + u\frac{\partial q}{\partial x} + v\frac{\partial q}{\partial y} + EP = 0 \qquad (4)$$

Both equations are very similar in structure and involve only advection (three dimensional) and either heating-and-cooling (HC) terms or evaporation-and-precipitation (EP) terms. The HC and EP terms are described in greater detail later.

The pressure tendency equation (alias the equation of continuity, alias the equation of conservation of mass) is

$$\frac{\partial p_\sigma}{\partial t} + \frac{\partial}{\partial \sigma}(\dot{\sigma} p_\sigma) + \frac{\partial}{\partial x}(u p_\sigma) + \frac{\partial}{\partial y}(v p_\sigma) - \frac{p_\sigma v}{r} \tan\Phi = 0. \tag{5}$$

The equation has three-dimensional mass divergence terms plus a metric term. The value of p_σ is a measure of the mass between the σ surfaces involved.

Finally, the hydrostatic equation is

$$g \frac{\partial z}{\partial \sigma} + c_p \theta \frac{\partial \pi}{\partial \sigma} = 0 \tag{6}$$

and expresses the balance of vertical forces.

3. VERTICAL STRUCTURE

The definition of the vertical σ coordinate and the vertical 9-layer structure of the model go hand-in-hand. Consider any two surfaces in the atmosphere where the pressure is known for all x and y (or λ and Φ) points, for example, the surface of the earth and the tropopause. The σ coordinate then is defined as the fractional distance (with "distance" in pressure units) between the upper pressure surface p_U to the lower surface p_L. Thus,

$$\sigma = \frac{p - p_U}{p_L - p_U} \tag{7}$$

where σ is nondimensional. It has the value 1 at $p = p_L$ and the value 0 at $p = p_U$. If we, for example, set $\sigma = 1/2$ a σ surface is defined that exists halfway between the tropopause and the surface of the ground and therefore shares in the variations of both height and pressure of both of those defining surfaces.

Note that p_σ ($= \partial p/\partial \sigma$) just equals $p_L - p_U$ as a consequence of (7). Thus p_σ is quite properly spoken of as the "pressure thickness" and this pressure thickness is just the quantity that is forecast by equation (5).

The model then uses the definition of σ from (7) to specify just where the nine layers are. Two p_L/p_U pairs are used: one defines the tropospheric σ domain in which p_U is the tropopause pressure p** and p_L is the surface pressure p*; the other pair defines the stratospheric σ domain wherein p_U = 50 mb everywhere (a constant pressure as well as constant σ surface) and p_L = p**. (See figure 1.) The tropospheric domain is divided into six equal pressure layers by defining σ surfaces to exist at σ_T = 1/6, 2/6,...,5/6 and the stratosphere is divided into three σ layers, making a total of nine layers in which our forecast variables are specified. The moisture, q, is carried only in the five lowest layers; elsewhere the model is dry. Physical boundary conditions are put on the model by requiring $\dot{\sigma} = 0$ at levels 10, 7, and 1. Physically, this states that no mass (air) can pass into the ground, thru the tropopause, or into or out of the stratosphere and thetasphere.

Earlier versions of the 8-layer model incorporated a surface boundary layer of fixed pressure thickness, p_σ, as the 7-layer model has. This was discarded in favor of increasing the resolution throughout the troposphere with the six equal tropospheric layers. The bottom layer still serves as a "boundary layer" as far as surface effects are concerned. The particular vertical structure shown in figure 1 is not to be considered absolutely fixed. Experience and experimentation may dictate changes in the layering structure; for example, a layer might be moved from the troposphere to the stratosphere or the tropospheric layers might be made unequal in thickness thus increasing resolution by making thinner layers near critical regions such as the ground or tropopause. Such alternations will be documented in future Technical Procedures Bulletins.

4. HORIZONTAL STRUCTURE

In the horizontal structure the data are carried on a regular 2.5° longitude-latitude grid (with no elimination of points or other artifical devices near the poles). At the poles all of the latitude-longitude intersection points refer to the same geographic point, hence the scalar variables θ, p_σ, q and others derived from them (π, Z, etc.) will be the same at all the pole points, while the one vector wind at the pole is resolved into the u and v components appropriate to the meridian along which one would approach the pole.

5. INITIAL CONDITIONS

The initial conditions for the forecast model can come from two sources. The primary source, in the routine analysis and forecast cycle, is the Optimum Interpolation (O/I) analysis. In that the results of this analysis are entirely within σ-coordinates and on the 2.5° grid nothing additional needs to be done. The model merely swallows up the O/I analysis and proceeds to forecast from it.

In those cases where an O/I analysis is not available (re-runs of pre-O/I implementation cases, special tests, etc.) it is necessary to revert back to the Hough analysis. The end results of the Hough function analysis are heights, winds and temperatures on the 12 mandatory levels up to 50 mb plus surface temperature, tropopause pressure and relative humidity on the six mandatory levels up to and including 300 mb. These data are on the 2.5° grid over the entire globe (10875 points per field). Transferring this information to the nine sigma layers of the model is done in essentially the same manner as in the 6-layer model described by Shuman and Hovermale (1968).

6. ADDITIONAL PHYSICAL EFFECTS

a. Friction

The model senses the ground by incorporation of a frictional drag term. There is no other friction in the model, only a surface friction term. The expression is based on the Prandtl layer theory and takes the form

$$\left.\begin{aligned} F_x &= \frac{g\rho}{\Delta p} C_D \left|\vec{V}\right| u \\ F_y &= \frac{g\rho}{\Delta p} C_D \left|\vec{V}\right| v \end{aligned}\right\} \qquad (8)$$

where Δp and ρ are the pressure thickness and mean density, respectively, of the lowest model layer, $\vec{V}$ is the vector wind (with u and v the components) in that layer, and C_D is the drag coefficient. The drag coefficient varies with the nature of the terrain and an explanation of how it is evaluated can be found in Cressman (3). The 9L GLO model uses the same C_D as the 7L PE does except of course on a λ-Φ grid and with a new set of values derived for the Southern Hemisphere.

b. Short Wave (Solar) Radiation

At any moment during the forecast we know, by calculation, what the solar zenith angle, ξ, is at each grid point. This knowledge plus a simple calculation of the precipitable water W given by

$$W = \frac{\Delta p}{g} q \qquad (9)$$

for the layer with pressure thickness Δp and forecast specific humidity q enables us to compute what the radiation people call the path length

$$u = W \sec \xi \qquad (10)$$

which represents the "amount," in some convenient units, of water vapor that the beam of solar radiation passes through and is at the same time partially absorbed by. Published computations exist (4) that relate path length to the actual quantity of energy absorbed from a beam; so it is a straightforward task to follow a sunbeam down through the lowest five layers of the model (the only ones that contain water) and calculate how much energy is absorbed and thus how much the layer is heated. This is part of the HC term of equation (3).

The previous discussion applies to clear sky conditions. If the descending beam hits a cloudy layer (which for present purposes is defined to be a layer with 90 percent humidity or greater), two things happen: 1) a fair portion of the beam is reflected back upward from the top of the layer thus causing less radiation to be available for subsequent absorption below (cloudy days are cooler days) and 2) both the reflected and transmitted radiation become diffuse and equation (10) no longer applies directly. Instead we use

$$u = 1.66\ W \qquad (11)$$

where 1.66 is an equivalency sort of a term, known as the Elsasser diffusion factor, enabling us to use beam absorption calculations even though the radiation is really diffuse. The now depleted and diffuse radiation proceeds through the cloud layer and any clear layers below being further absorbed as it goes. Any further encounters with clouds will cause additional upward reflection but no additional diffusion. Once diffuse, always diffuse. When the radiation reaches the ground, it undergoes a final reflection and we proceed to calculate the further absorptions of the upwelling radiation streams. These streams consist of the radiation reflected both from the ground and any cloud layers met on the way down.

The albedos (the ratio of reflected to incident radiation) for the possibly cloudy layers from the ground up are, at present, 0.7, 0.6, 0.5, 0.4, and 0.3; that is, low clouds reflect a greater portion of what hits them than high clouds. The albedo of the ground is a geographic variable. All sorts of thermal things happen at the ground besides reflection of solar radiation--these are detailed shortly.

The 7L PE model incorporates a portion of these shortwave calculations--in effect the heating from the downward coursing stream was included but not the upward stream heating. It's a small refinement but one that was easy to incorporate.

c. Long Wave (Terrestrial) Radiation

Calculation of the heating and cooling due to long-wave radiation is somewhat more involved (and for that reason was not included in the 7L PE model) since radiant energy is both emitted and absorbed in situ by the water vapor of the model atmosphere itself.

The computational procedure is one of determining the radiative flux at each level (interfaces between layers) by adding up the radiation emitted from each adjacent layer, plus the radiation emitted from the layers once removed less the portion of that radiation absorbed in the adjacent layers, plus the radiation emitted from layers twice removed less the absorption in the two intervening layers and so on and on. The procedure does not reach out forever, of course. It will terminate at the top of the moisture containing portion of the model (above layer five), at the edge of a cloudy layer (90 percent humidity or more) or at the ground. A cloud layer is considered a "black body" (in radiation terms). All infrared radiation hitting it is completely absorbed, none passes through, and its radiative emission depends only upon its temperature. Thus, the summation described above stops at a cloud top or bottom. The ground is also a black body. Once all the fluxes, both upward and downward at all levels, are computed it is a simple task to ascertain the net flux in each layer. This flux is the net energy gain or loss of the layer and thus the temperature change, another contribution to the HC term of equation (3) can be found.

For those with a particular interest in the matter, the flux calculations are made by integrating the wavelength integrated emissivity gradients times the black body emission over the path length. We are using emissivities derived by Kuhn (5).

d. Surface Energy Exchanges and Temperature

We are introducing some new physical effects to the 9L GLO model to describe, hopefully, the various energy exchanges at the ground (and ocean) surface with the end result of being able to make a forecast of surface temperature. These effects are present in the 7L PE only in a very emasculated form and are not capable of producing surface temperatures.

The method and underlying physical assumption is to compute all the various forms of energy flux through the surface of the ground, in either direction, and require that the resultant flux (the net flux) shall be zero. The various surface energy flux terms and formulae for computation are:

1) Net short wave radiation. This is a portion of the calculation made in section 6. b., and is simply the downward flowing radiation less that reflected at the ground.

2) Downward streaming long wave radiation. This, similarly, is a portion of the calculations from section 6. c., found by the summation procedure described there.

3) Sensible heat flux. The expression is

$$H = \rho \left| \vec{V} \right| C_D c_p (\theta - \theta^*). \qquad (12)$$

The density ρ, wind $\vec{V}$, and potential temperature θ are those of the lowest model layer, and θ^* is the ground or sea surface temperature. This expression allows for heat flux in either direction over land or water thus incorporating heating or cooling of air over land and cooling over water, three effects not included in the 7L PE model. Note that we are using equation (12) in two capacities. It is both part of the surface flux calculation (our present concern) and the heat energy passing into or out of the lowest layer of the model which contributes to temperature changes in that layer--a further portion of the HC term of equation (3). A special note: if H is positive, a statically stable situation, its value (and quantities associated with it) is reduced by a factor of 10.

4) Latent heat flux. The expression is

$$LH = \rho \left| \vec{V} \right| C_D L (q - w q^*_s) \qquad (13)$$

which is obviously similar to the sensible flux expression, except here we have the latent heat of evaporation L and the specific humidity. The saturation specific humidity at the surface is q_s* which is a function of the surface temperature and pressure only. The "potential evaporation," w is a sort of ground wetness parameter described by Saltzman (6). Over the oceans w = 1, while over land w will be less than one (but always greater than zero) expressing the physical effect of land surfaces having a less than complete availability of water for evaporation purposes. In the present model, we are setting w equal to one minus the albedo, i.e., assuming that low albedo areas (forest lands) are relatively moist and conversely (i.e., deserts are dry). As with equation (12), the latent flux can go either away from the surface (evaporation) or toward the surface (dew formation). Equation (13) is, like (12), also used in a double capacity both for the flux determination and, with the latent energy flux converted to amount of moisture units, as a contribution to the EP term of equation (4) for the lowest layer moisture tendency. Thus the 9L GLO model includes evaporation over land and water surfaces (and dew formation, too) while the 7L PE model included only evaporation over water as a source of moisture for the model atmosphere.

5) Energy flux into or out of the ground. Some sort of explicit ground storage source/sink term could be incorporated into the model if we wanted to go into the geology business. For the time being we don't and instead we will assert that the effect of the ground storage will be such as to ameliorate any changes in the surface temperature computed without reference to the ground flux term. The ground term will by implication always reduce surface temperature changes, and will always be a brake.

6) Upward Streaming Long Wave Radiation. This is really the leftovers term which is the term used to balance out the other four flux terms to get a net flux of zero. The black body radiation from the ground is

$$LW_g = \sigma(T_E^*)^4 \tag{14}$$

where σ is the Stefan-Boltzman constant, and T_E^* the Kelvin scale surface temperature. The fluxes of paragraphs 1-4 are summed, set equal to LW_g and equation (14) is solved for T_E^* to give the equilibrium surface temperature. The ground storage effect is then introduced by computing a tendency for the surface temperature δT^*. This temperature is given by

$$\delta T^* = \beta(T_E^* - T^*) \tag{15}$$

where T^* is the surface temperature from the previous time step of the forecast and β(= 0.05) specifies the amount of lag, "caused" by the "ground storage," in the surface temperature change. Over open water β = 0, i.e., the sea temperature remains constant throughout the forecast.

7. PRECIPITATION FORECASTS

The large scale or saturation rain is dependent upon the forecast values of q and a forecast value of the saturation specific humidity q_s. The latter is a function of the forecast temperature and pressure in each layer. The q values are not compared directly against the q_s values but upon diminished values. The amount of the dimunition, or scaling, is a function of the σ-layer and temperature within the layer. Thus rain can fall from a layer of less than 100% mean relative humidity. This is as it should be as the layers in the model are quite thick relative to rain producing layers in the real atmosphere.

Whenever q is forecast to exceed the scaled q_s condensation occurs in such an amount as to bring q down to the scaled q_s value and a corresponding amount of latent heat is released. Most of the latent heat serves to warm the layer in which the supersaturation/condensation occurs; however, a fraction of the latent energy is placed in the lowest layer.

The rain does not fall undisturbed. Rain from a saturated layer falling into a relatively dry one will undergo partial evaporation and what is left will continue to the next layer, where the process may repeat.

One other source of moisture exists for the model atmosphere, evaporation from the ground. The latent heat flux either from the ground or to the ground, in the case of dew formation, is converted to flux of specific humidity and is a contribution to the q tendency equation for the lowest layer.

There also exists convective precipitation in the model dynamics. The subgrid convective parameterization is basically a series of tests of the instantaneous dynamic and thermodynamic conditions of the model. Then, depending upon whether the tests are satisfied, adjustments to the forecast quantities of the model are made in amounts related to the quantitative results of the tests.

Both large scale and convective precipitation are generated from forecasts made within the longitude/latitude boxes, prior to the longitudinal averaging of the tendencies. This has the double advantage that the precipitation forecast is made on the finest resolution available, while the possibly substantial latent heat effects still undergo the longitudinal averaging needed to discourage linear instability.

8. CONVECTIVE ADJUSTMENTS (Dynamic Stability Adjustment)

When a super-adiabatic lapse rate develops, both the atmosphere and the numerical models become unstable. The atmosphere resolves the instability by mixing in the vertical all by itself, so to speak. We have to tell the model what to do and naturally we follow the atmosphere's lead and induce vertical mixing between the unstable layers.

The procedure is quite straightforward: if two dry layers are found to be unstable, they are both reassigned a mass weighted mean potential temperature

$$\theta_m = \frac{\theta_U \Delta p_U + \theta_L \Delta p_L}{\Delta p_U + \Delta p_L}$$

where U and L refer to the upper and lower layers respectively. The test then repeats for the next pair of layers above using the newly adjusted temperature in what is now the lower of the two layers.

If a layer is saturated, it is tested for stability with respect to the layer above it using a saturation pseudo-adiabat and if found unstable, the two layers have their temperatures adjusted to lie on the appropriate pseudo-adiabatic temperature curve.

9. OUTPUT OF FORECASTS

After marching forward in time for many 10-minute time steps, the model atmosphere eventually reaches an appropriate hour for ourput. The forecasts that go out from NMC are for the most part not in σ coordinates but generally on constant pressure or constant height surfaces. (The few exceptions are such items as boundary layer winds and temperatures--and surface temperatures if they turn out to be any good--and tropopause related quantities.) It is necessary to interpolate from the σ coordinate to the pressure coordinate. This is accomplished in the same way as the 7L PE model--basically either a hydrostatic calculation of heights at known mandatory pressure surfaces given the heights, pressure and temperatures at surrounding σ surfaces, or a linear interpolation of $\vec{V}$, θ, or q (linear with π) from the σ layers to the pressure surfaces (2). The same special underground extrapolation to sea level (or 1000 mb) is carried in the 9L GLO model as is done in the 7L PE model (7).

10. REFERENCES

1. Technical Procedures Bulletin No. 105: "A Description of the Flattery Global Analysis Method--No. 1," January 10, 1974.

2. Shuman, F., and J. Hovermale, 1968: "An Operational Six-layer Primitive Equation Model," J. Appl. Meteorol., 7, 525-547.

3. Cressman, G. P., 1960: "Improved Terrain Effects in Barotropic Forecasts," Mon. Wea. Rev., 88, 327-342.

4. Manabe, S., and R. T. Wetherald, 1967: "Thermal Equilibrium of the Atmosphere with a Given Distribution of Relative Humidity," J. Atmos. Sci., 24, 241-259.

5. Kuhn, P. M., 1963: "Radiometersonde Observations of Infrared Flux Emissivity of Water Vapor," J. Appl. Meteorol., 2, 368-378.

6. Saltzman, B., 1967: "On the Theory of the Mean Temperature of the Earth's Surface," Tellus, 19, 219-229.

7. Technical Procedures Bulletin No. 57: "Revised Method of 1000 mb Computation in the PE Model," December 2, 1970.

Vertical Structure

Fig. 1 is a schematic diagram of the vertical structure of the 9-layer model with indications of the approximate pressures for the various σ surfaces.

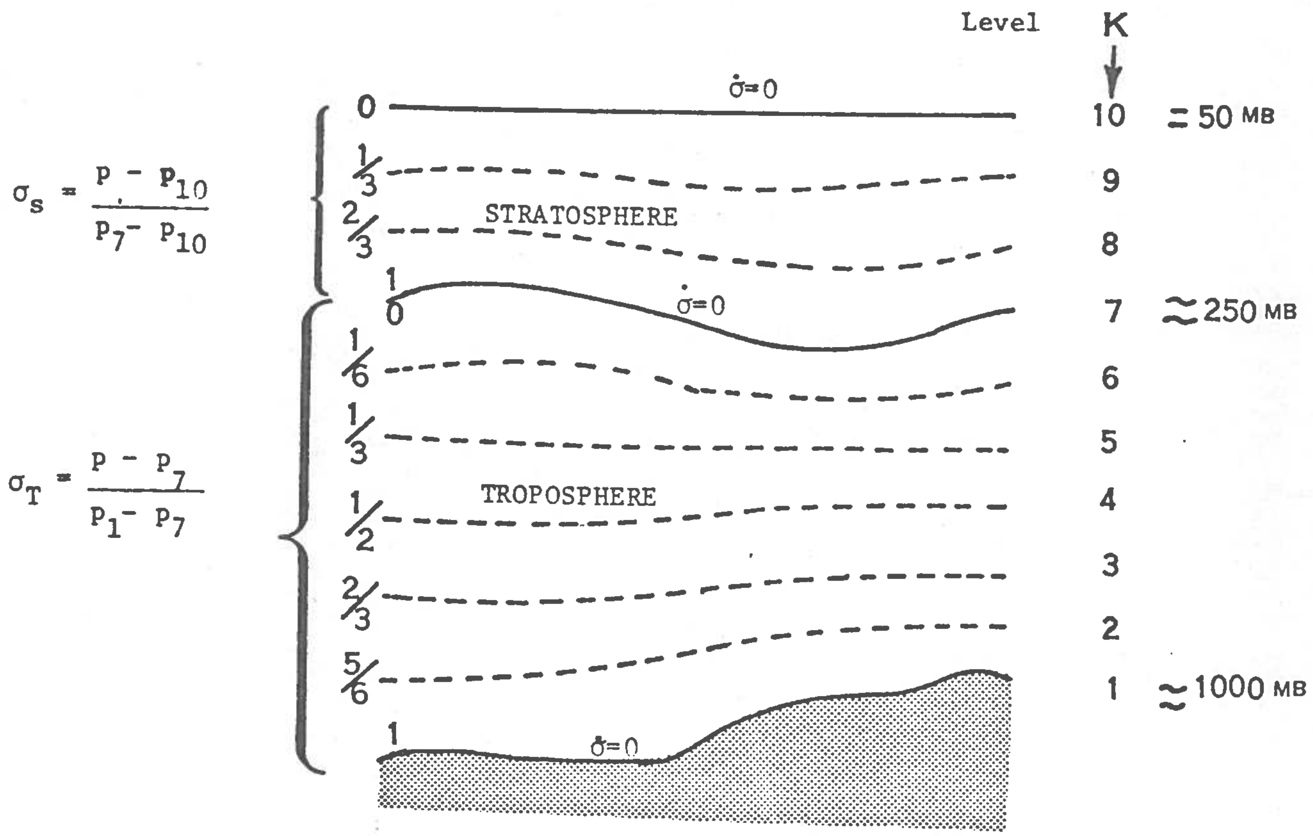

Fig. 1 Schematic Diagram of the vertical structure

Two σ domains are defined: a tropospheric and a stratospheric domain. The tropospheric domain is defined between the surface pressure, p_1, and the tropopause p_7:

$$\sigma_T = \frac{p - p_7}{p_1 - p_7}$$